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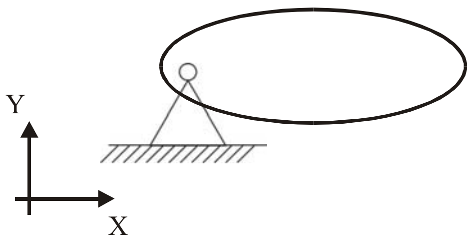
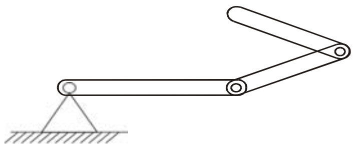
Problem 1:

Please fill in the number of dof.

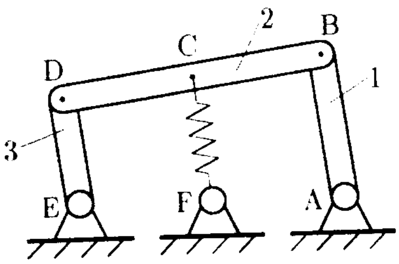
2-Dimensional

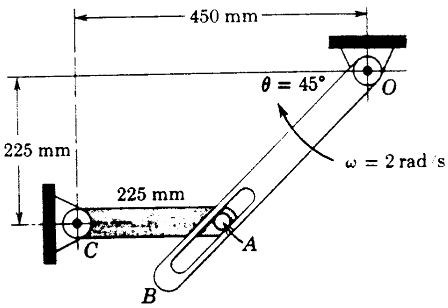




a) dof=

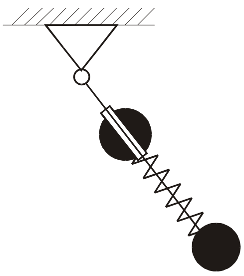
b) dof=





c) dof=

d) dof=



e) dof=

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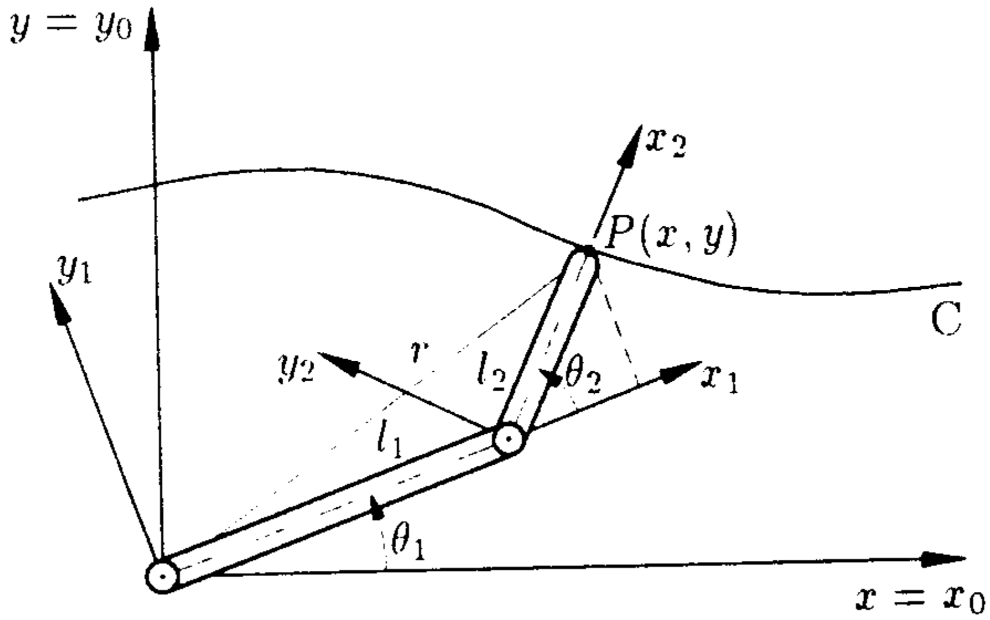
Problem 2:

Determine the workspace X of a 2-arm-hinge-mechanism.
The configuration space is given by:

$$Q = \left\{ \theta_1, \theta_2 \mid 0 \le \theta_1 \le \frac{\pi}{2}; 0 \le \theta_2 \le \pi \right\}$$

Display the workspace graphically.



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Problem 3:

Determine the velocity and the acceleration of a particle in cartesian coordinates described by the following coordinate vector:

$$\underline{q}(t)=\begin{bmatrix} r(t) \\ \varphi(t) \\ z(t) \end{bmatrix} \qquad \underline{k}(t)=\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

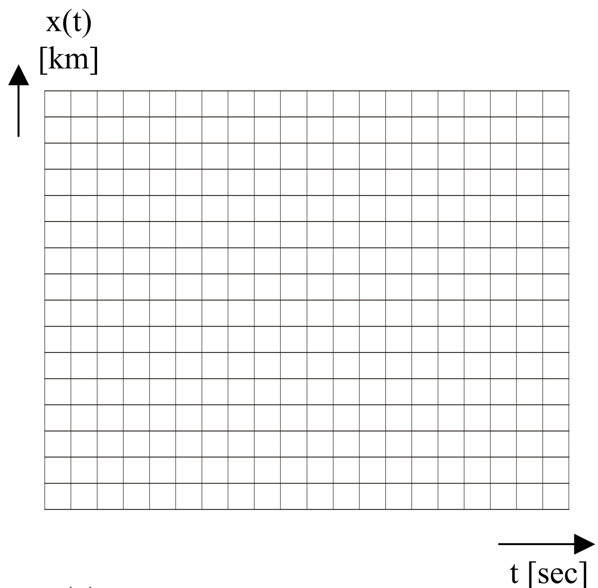
Note: Begin with the relationship between the cartesian coordinates $\underline{k}(t)$ and the cylindrical coordinates $\underline{q}(t)$.

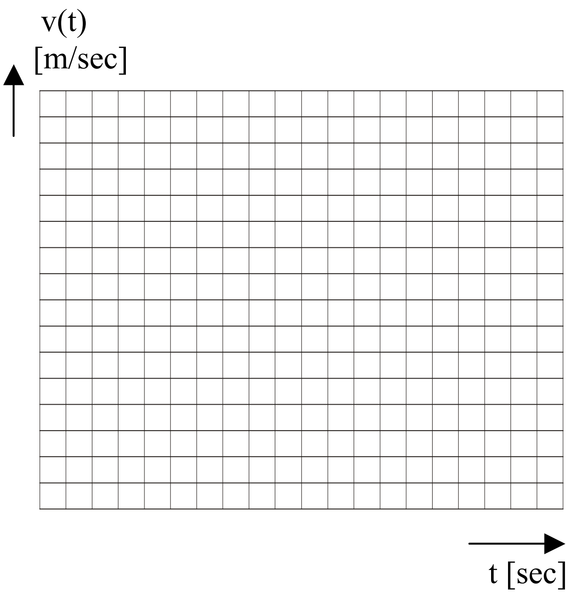
Problem 4:

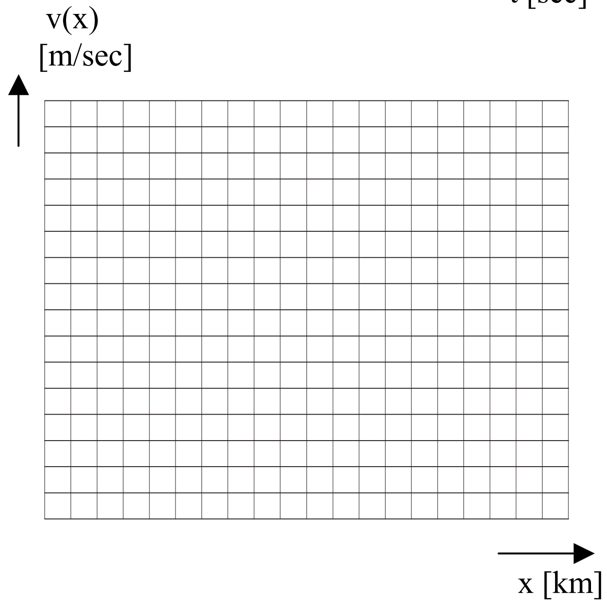
An ICE III train is constantly accelerated with a_0 until reaching the velocity $v_1 = 330\frac{km}{h}$.

The initial velocity is $v_0 = 0\frac{km}{h}$.

1. Determine the acceleration a_0 if the train should reach v_1 after $x_1 = 30km$?
2. Determine the time until the train reaches the velocity v_1.
3. Outline the curves in the prepared diagrams.







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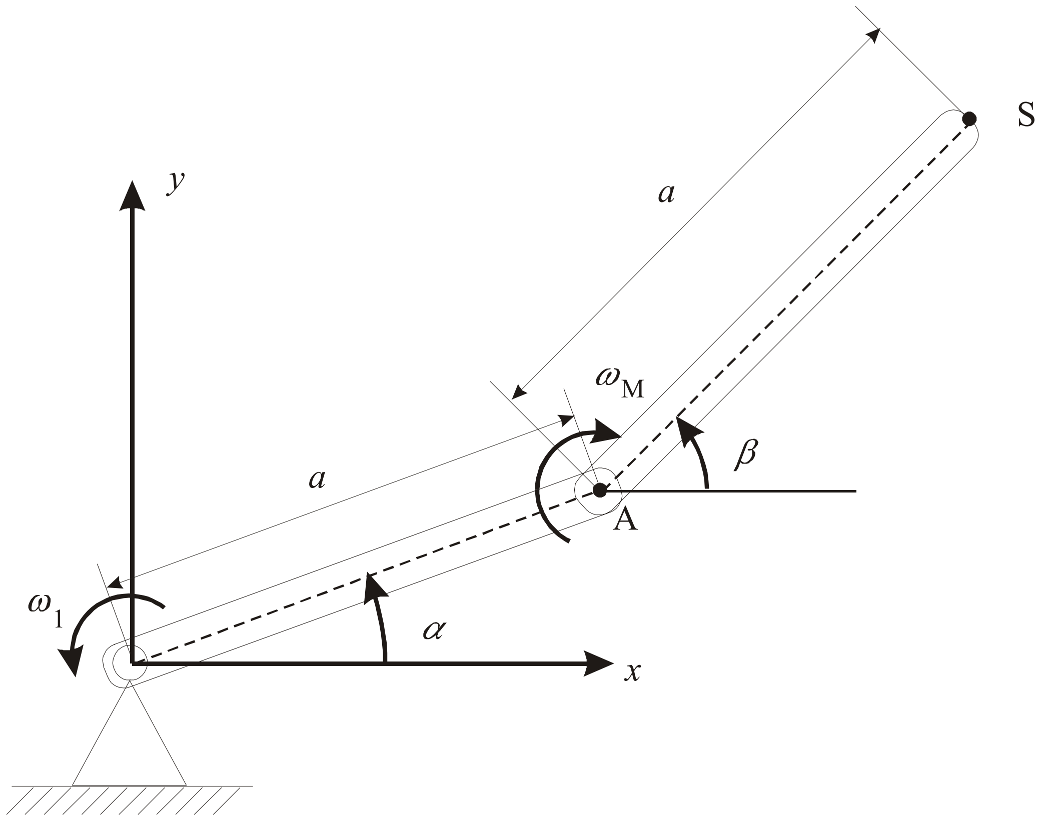
Problem 5:

A robot arm consists of two parts, which are simply supported in point A. The lower part rotates anti-clockwise with the angular speed ω_1 The upper part is powered clockwise by a flanged engine at the intermediate hinge (angular speed ω_M). Compute the path and the velocity of the tip S using angle α_0 and angle β_0 at the time $t=0$.



given: $\omega_1, |\omega_M| = 2 \cdot \omega_1, \alpha_0, \beta_0$

Problem 6:

Determine following equations.

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & 1 \\ 1 & 5 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 4 & 3 \\ 3 & 1 & 6 \\ 1 & 5 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -3 \\ -3 & 27 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$$

a) $A + B =$

b) $A - B =$

c) $A \cdot B =$

d) $A^T =$

e) $B^{-1} =$

f) $\left(\underline{\underline{C}} - \lambda \, \underline{\underline{D}} \right) \underline{\varphi} = 0$

g) $\sin(\alpha \pm \beta) =$

$\cos(\alpha \pm \beta) =$

h) $\sin(30°) =$

$\cos(30°) =$

$\sin(60°) =$

$\cos(60°) =$

$\sin(45°) =$

$\cos(45°) =$

i) $E^T =$

j) $E^{-1} =$

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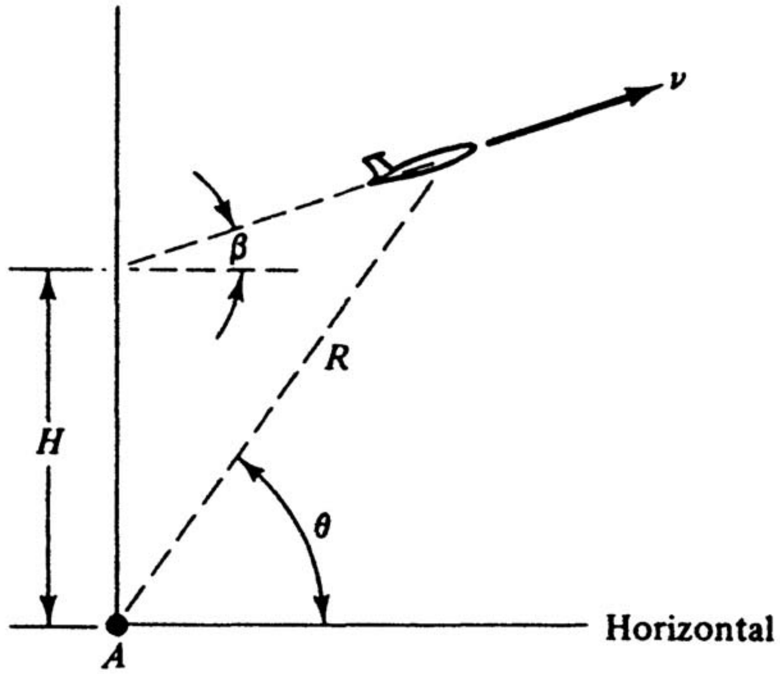
Problem 7:

An airplane climbs at a constant speed *v* and at a constant climb angle *β*. The airplane is being tracked by a radar station at point *A* on the ground.

Determine the radial velocity $\dot{R}$ and the angular velocity $\dot{\theta}$ as functions of the tracking angle θ.



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Problem 8:

Use the concept of a joint kinematical description to determine $\dot{R}$ and $\dot{\theta}$ for the airplane in Problem 7.

Problem 9:

Determine the velocity and the acceleration of a particle in cartesian coordinates described by the following coordinate vector:

$$\underline{q}(t) = \begin{bmatrix} r(t) \\ \varphi(t) \\ z(t) \end{bmatrix}$$

Note: Begin with the relationship between the cartesian coordinates $\underline{k}(t)$ and the cylindrical coordinates $\underline{q}(t)$. Use the Jacobian matrix and the matrix $\underline{\underline{K}}_{rq}$ to determine the velocity and the acceleration respectively.

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Problem 10:

Using the chain rule of differentiation we get.

$$\underline{a}(t) = \underline{\ddot{r}}(t) = \underline{\underline{J}}_{rq}\, \underline{\ddot{q}}(t) + \underline{\underline{K}}_{rq}\, \underline{\dot{q}}_{q}$$

Determine $\underline{\underline{K}}_{qr}$ and $\underline{\dot{q}}_{Q}$.

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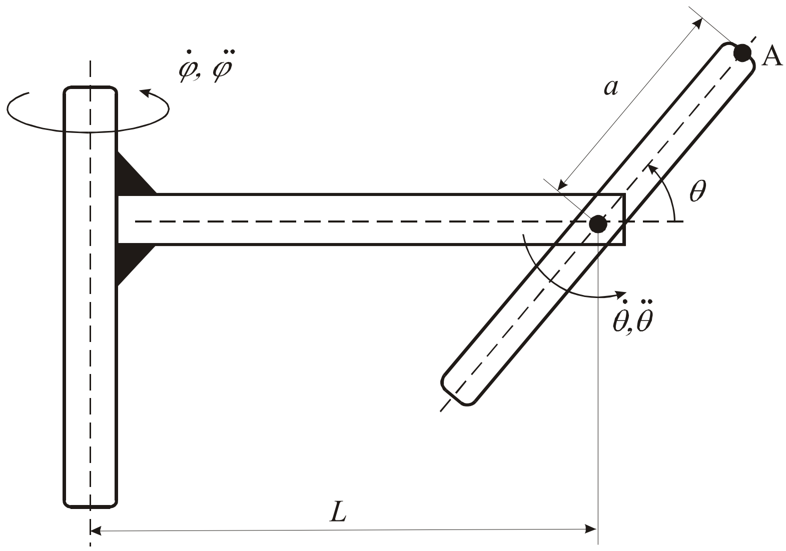
Problem 11:

A vertical standing bar is rotating with the angular speed $\dot{\varphi}$ about his longitudinal axis. A second bar with length L is welded perpendicular to the axis of rotation. A third bar is connected by a hinge (see figure) on his free end. Determine the position vector, the velocity vector and the acceleration vector of point *A*.



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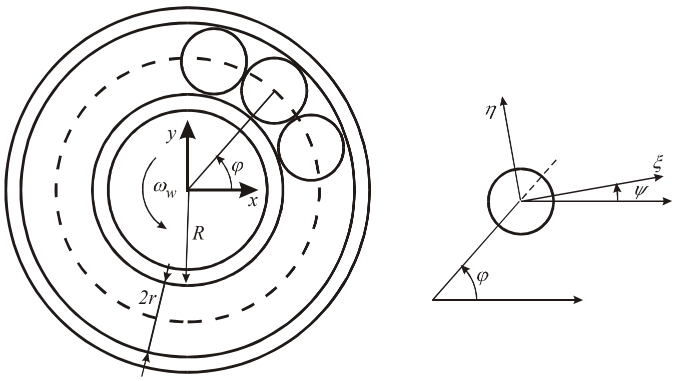
Problem 12:

A roller bearing consists of *n* cylinders (radius *r*), an outer race fixed on the case and an inner ring that is heat-shrunk on a shaft. The shaft is rotating anti-clockwise with angular speed ω_W. The cylinders are rolling on the inner ring and the outer race without slipping. The tangency between the cylinders, the inner ring and the outer race can be assumed to be a line geometry (*R*>>*r*). A contact between the cylinders is structural prevented.



1. Determine the angular speed ω_Z of the cylinders rotating about their own midpoint.
2. Determine the angular speed ω_C of the cylinders rotating about the shaft.
3. Determine the frequency a faulty point on the outer race is overrun.

Given: r, R, ω_W, n, $\varphi(t=0)=0$, $\psi(t=0)=0$

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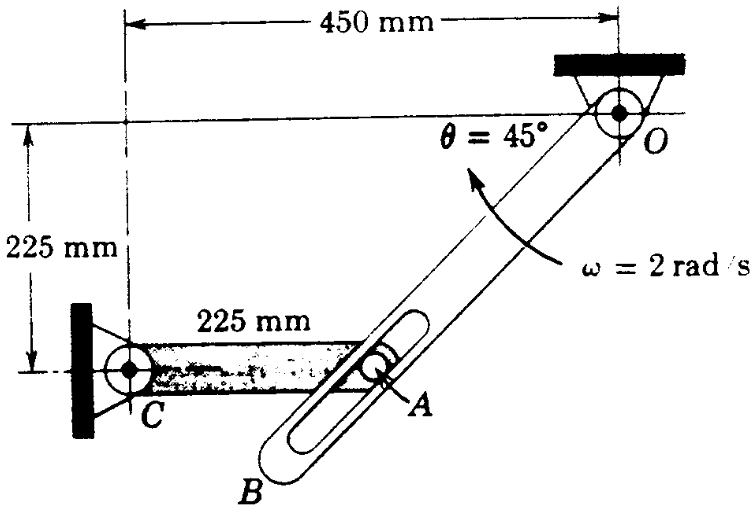
Problem 13:

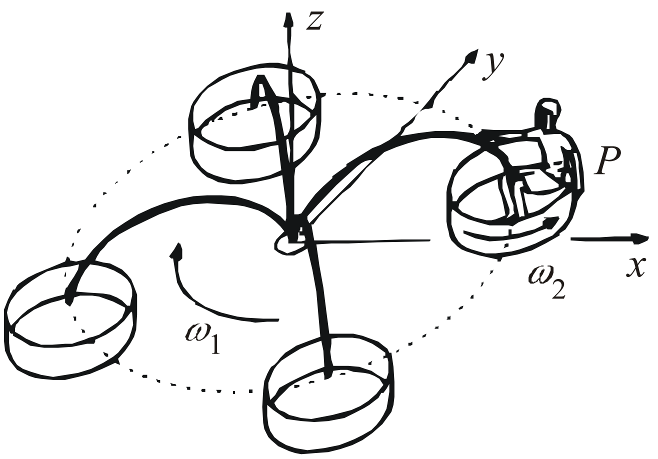
Two rods, which are simply supported at point *O* and *C*, are connected by a slide ring that is guided in a link at point *A*. The upper rod rotates with angular speed ω = 2 rad / sec about point *O*. Determine the relative velocity between the slide ring and the link. Consider especially the case that θ is equal to *45* degrees!

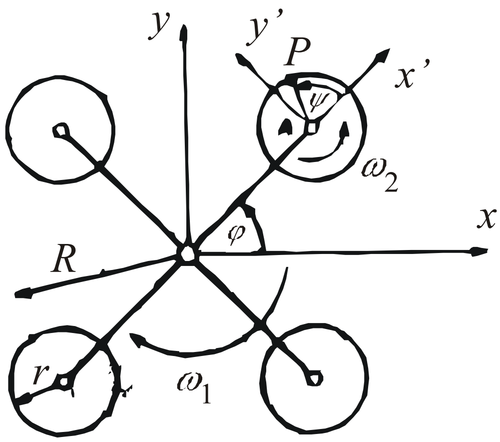


Problem 14:

A person is riding a roundabout, which is rotating clockwise with angular speed ω_1. A cage is mounted on every arm. The cages are powered anti-clockwise by flanged motors with angular speed ω_2.

1. Determine the absolute velocity of *P* in the inertial system ${}_I\underline{v}$.
2. Determine the absolute acceleration of *P* in the inertial system ${}_I\underline{a}$.
3. Determine the relative velocity of *P* with respect to the arm ${}_R\underline{v}$.
4. Determine the absolute velocity of *P* in the *K1-coordinate system*.





Given: $r, \quad R = 3r, \quad \omega_1, \quad \omega_2 = 4\omega_1, \quad \varphi(t=0) = 0, \quad \psi(t=0) = 0$

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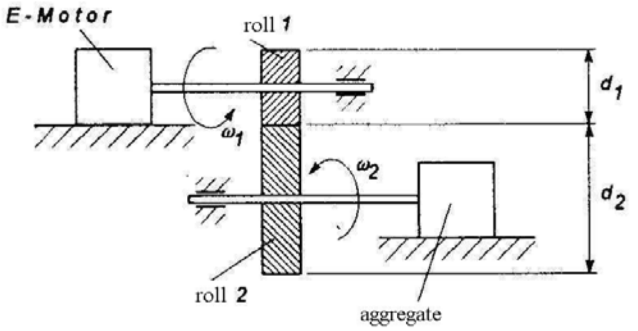
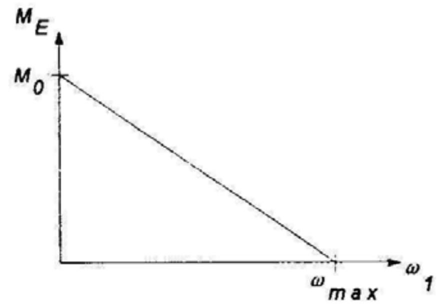
Problem 15:

An electro motor that delivers an angular speed dependent drive moment powers an aggregate by a gearbox without sliding. The aggregate has a constant load moment M_L. The gears (German: Zahnräder) have the masses m_1, m_2 and may be assumed to be cylindrical rolls with diameters d_1 and d_2. The mass moment of inertia of the electric motor is J_E and of the aggregate is J_A. The mass moments of inertia of the shafts can be neglected.





Given: Gears: m_1 , d_1 , m_2 , d_2 ; Aggregate: J_A , M_L ; Motor: J_E ;
$M_E = M_0 - k\omega$; $M_0 = const. > 0$, $k = const. > 0$

1. Determine the angular speed of the electro motor run up of a halt.
2. Determine the maximum angular speed of the electro motor at the load moment M_L.
3. Determine the drive moment at a steady state.
4. Determine the conditions which must be fulfilled for M_0.
5. Determine the variation of the aggregate angular speed if the gear transmission ratio $\left(\frac{d_1}{d_2}\right)$ is doubled.

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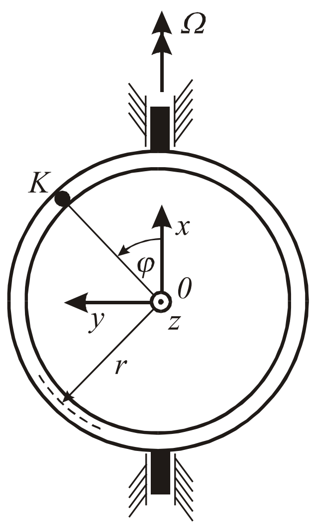
Problem 16:

A ring with medial radius r is rotating with constant angular speed ω about the x-axis. The ring consists of a free movable sphere. The acceleration due to gravity can be neglected.

1. Determine the acceleration of the sphere K in the inertial system.
2. Determine the relative acceleration of the sphere K with respect to the ring.
3. Determine the Coriolis acceleration of the sphere K in the inertial system.



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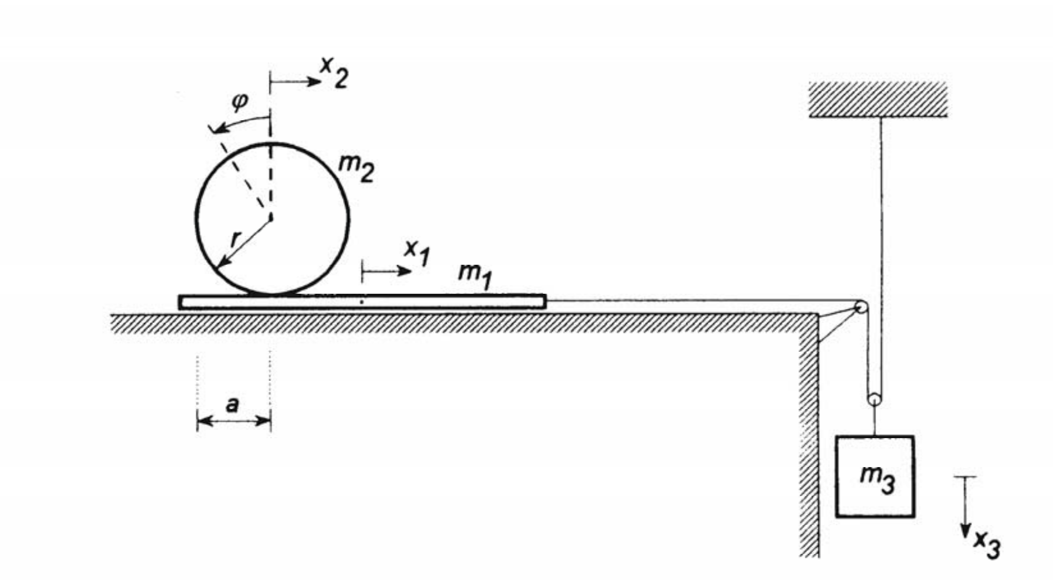
Problem 17:

A homogeneous cylinder (mass m_2, radius r) is lying on a thin shelf (mass m_1). A rope is fixed to the thin shelf, rolling over two pulleys and fixed to an attachment point (see figure).
Mass m_3 is fixed to the centre of gravity of the second pulley. The inertial velocity of mass 3 is $\dot{x}_3 = 0$. The figure shows this inertial condition. If mass 3 is no longer fixed the system starts to move. The cylinder is rolling on the thin shelf and after a certain time it is falling off. Assume that there is no friction between the shelf and the underlay and no sliding between the cylinder and the shelf. Neglect the mass of the rope and the pulleys.



1. Determine the acceleration of the shelf.
2. Determine the distance a when the cylinder is falling off the shelf.

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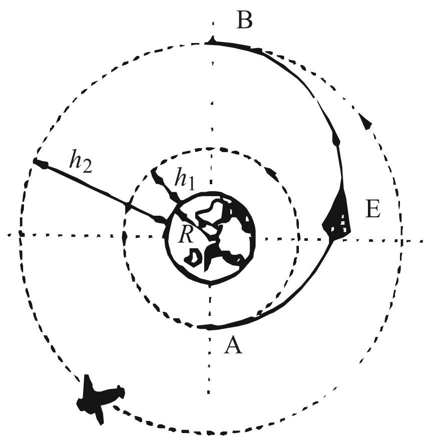
Problem 18:

A space vehicle moving in a circular orbit of radius r_1 transfers to a larger circular orbit of radius r_2 by means of an elliptical path between *A* and *B*. The transfer is accomplished by a burst of speed Δv_A at *A* and a second burst of speed Δv_B at *B*. The burning time is negligible smaller than the time of circulation.

1. Write expressions for Δv_A and Δv_B in terms of the radius and highs shown and the value *g* of the acceleration due to the gravity at the earth's surface.
2. Determine the burning time t_A and t_B to achieve the desired changes of velocity.

Given: R = 6370km; h_1 = 900km; h_2 = 1200km; g = 9,81m/s^2; m = 4000kg; F_s = 10kN



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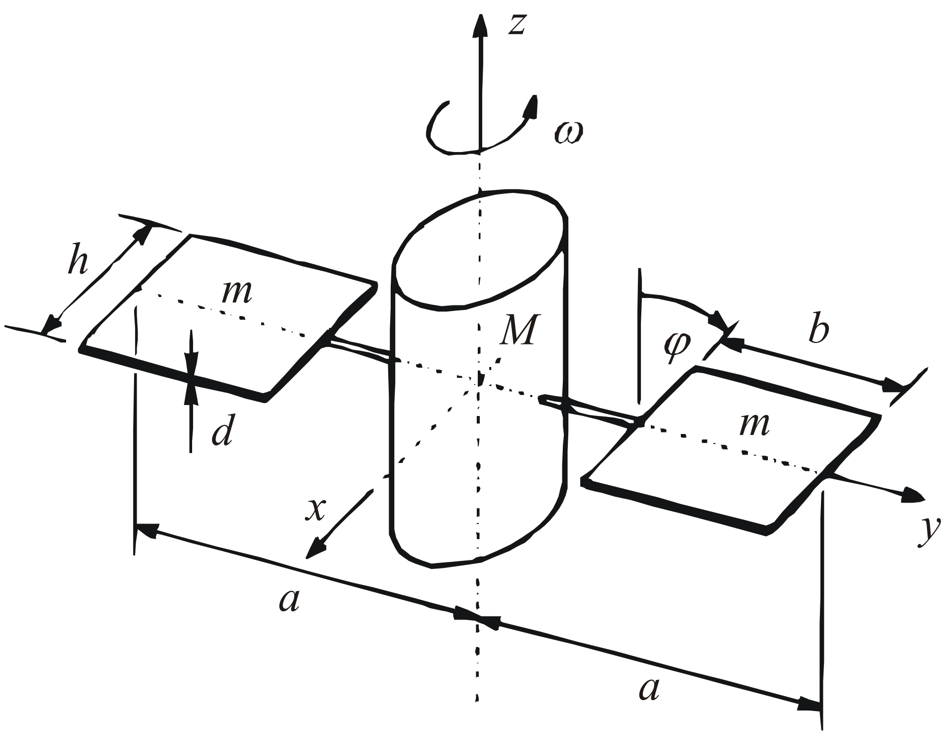
Problem 19:

A satellite consists of a rotationally symmetric satellite body (mass *M*, radius of gyration *i*) and two identical solar panels (mass *m*, dimension *b*, *h*). The two solar panels may be accepted as two thin rectangular plates with thickness *d*. The satellite is rotating about the *z*-axis with angular speed ω if the solar panels are at the position $\varphi = 0°$. The solar panels can be twisted about the *y*-axis using a mechanism.



Given: M = 160kg; i = 0,45m; a = 2,5m; m = 8kg; b = 1,8m; h = 1,2m; d << b; $\omega = 5\,s^{-1}$

Determine the angular speed $\omega_{90°}$ if the solar panels are at the position $\varphi = 90°$.

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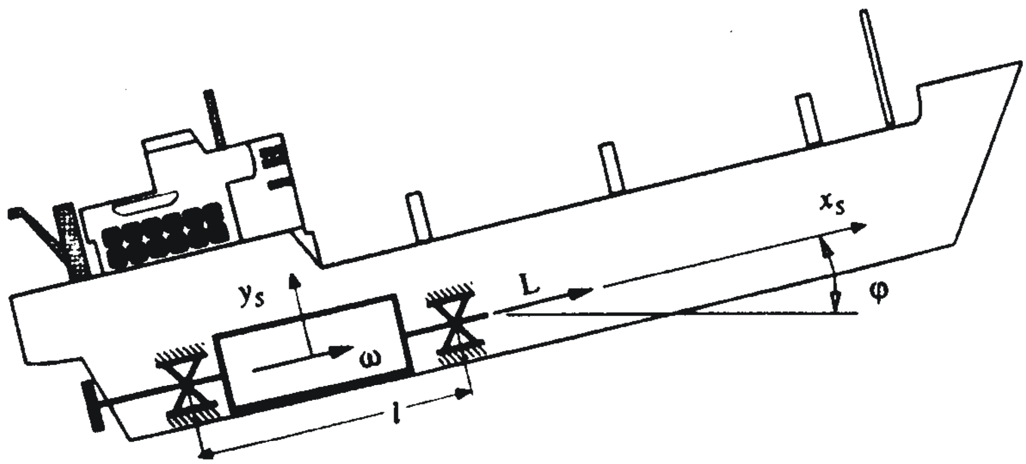
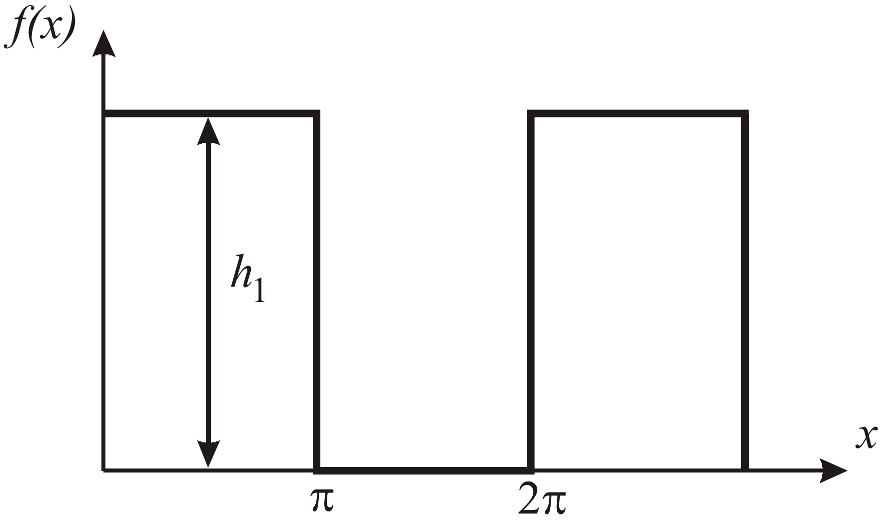
Problem 20:

Determine the maximum produced force to the bearings of a power turbine due to the gyroscopic forces. The ship is pitching with an amplitude of 9° and a time of circulation of 15 sec. about an axis that is perpendicular to the rotor axis. The mass of the power turbine is m = 3500 kg, the radius of gyration is k = 0,6 m and its rotational speed is n = 3000 $\frac{1}{\text{min}}$. The bearing distance is l.



Problem 21:

Determine the Fourier coefficients a_k, b_k of the diagrammed function.
Determine the first five coefficients of the Fourier series.



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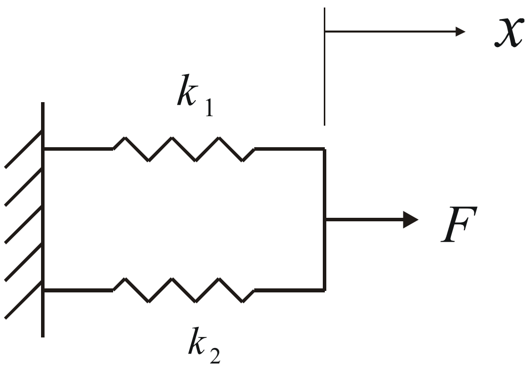
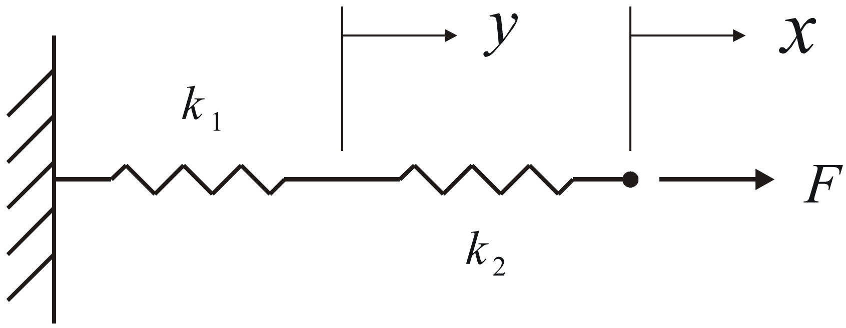
Problem 22:

Find the equivalent spring constant for the systems shown in the two figures.

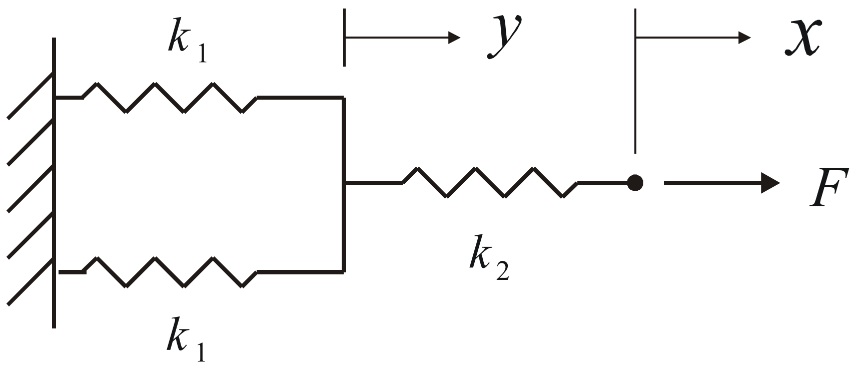
a) System consisting of two springs in parallel



b) system consisting of two springs in series

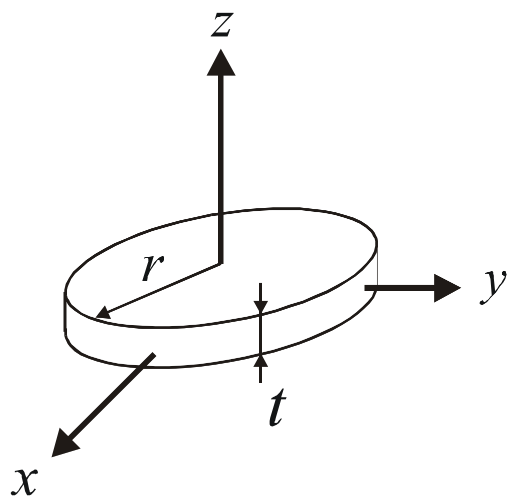


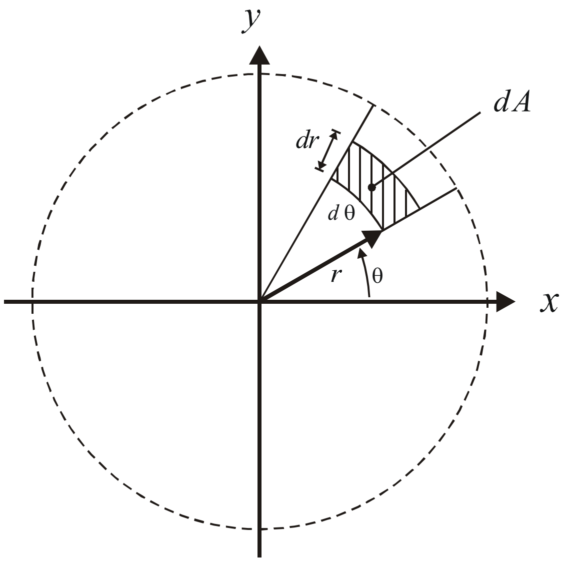
c) system consisting of three springs in parallel and in series



Problem 23:

For the homogeneous disk of mass *m* and radius *r* shown, calculate the moments of inertia about the *x* axis and *z* axis. Assume that the origin of the *xyz* coordinate system is at the centre of gravity and that the disk is symmetrical about each axis.





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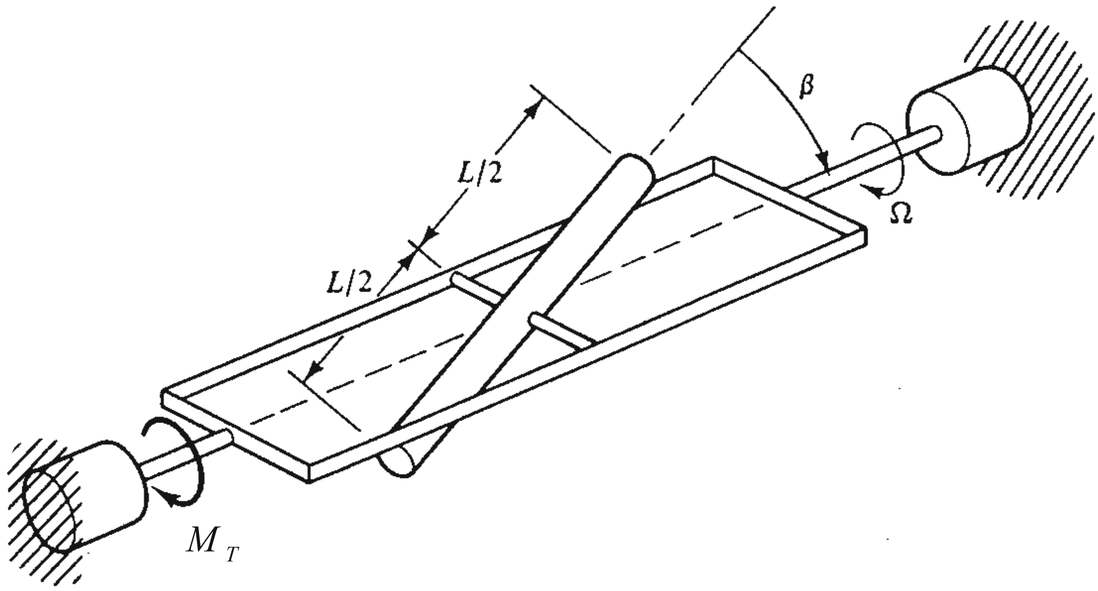
Problem 24:

The slender bar is mounted on a gimbal that rotates about the horizontal axis at constant rate Ω due to torque M_T. Derive the differential equation governing the angle β between the bar and the horizontal axis, and also derive an expression for M_T.



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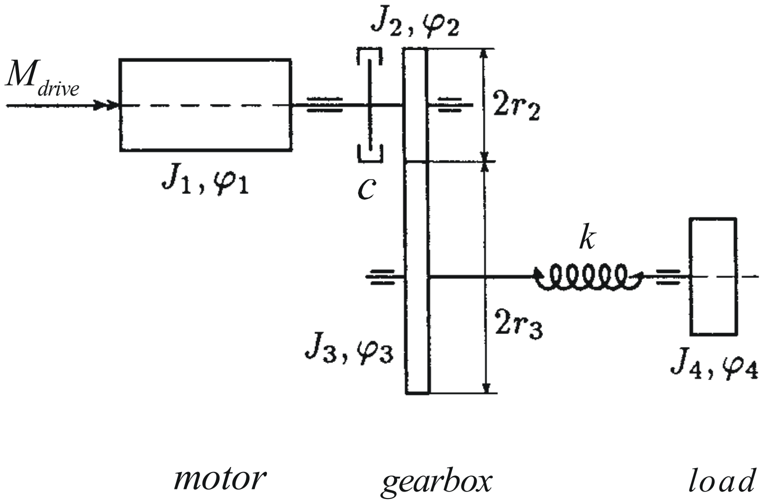
Problem 25:

Determine the differential equation of motion of the gearbox shown in the figure using Lagrange's Equations of Motion:



k: stiffness
c: angular speed dependent damping

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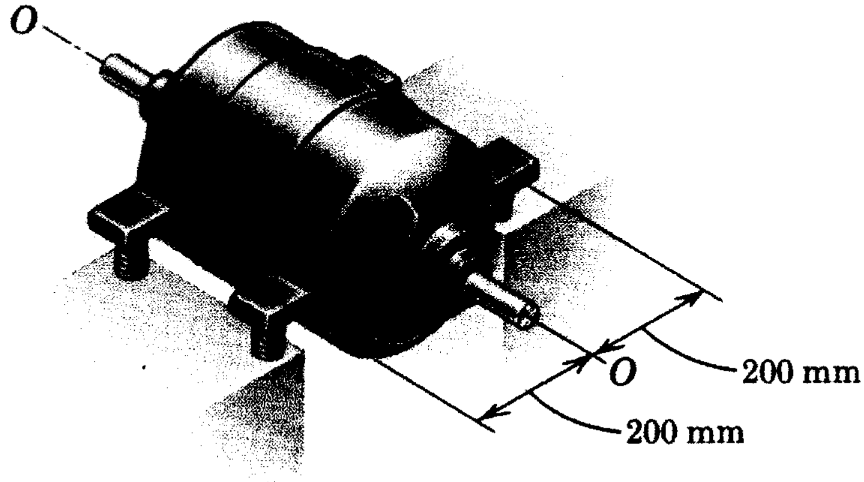
Problem 26:

If you slowly increase the rotation speed of the figured motor a vibration oscillation of the whole motor occurs at a rotation speed of $360\ min^{-1}$ about the 0 – 0 axis. This shows that this rotation speed is identical to the eigenfrequency of the motor. Determine the stiffness k of each of the four identical springs, if the motor has a mass of $43\ kg$ and a radius of gyration of $100\ mm$ about the 0 – 0 axis.



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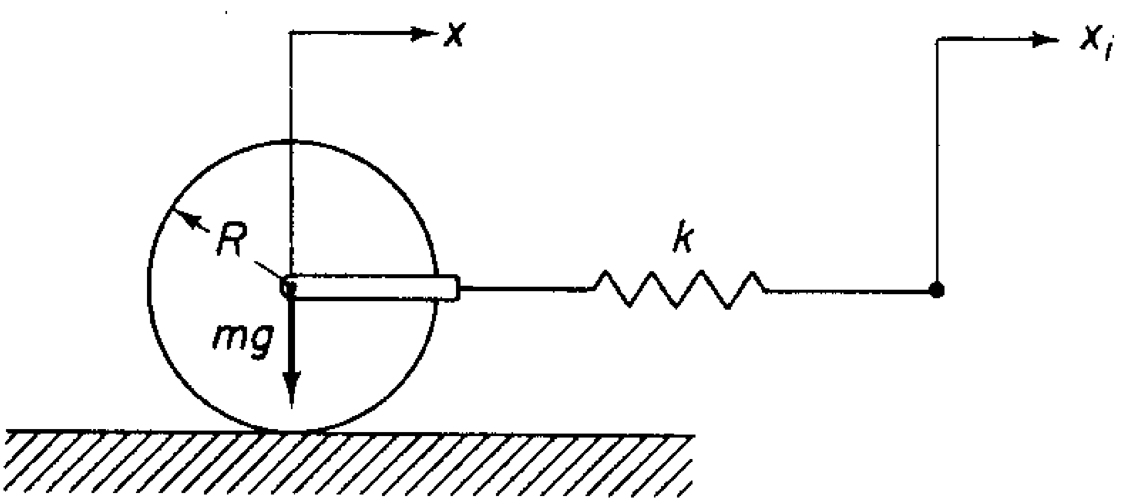
Problem 27:

Consider the system shown, where the cylinder of radius R and mass m is pulled through a massless spring with spring constant k. Assume that the cylinder rotates freely about its axis and the input displacement x, is known. What is the natural frequency of the system? Assume that there is no sliding.



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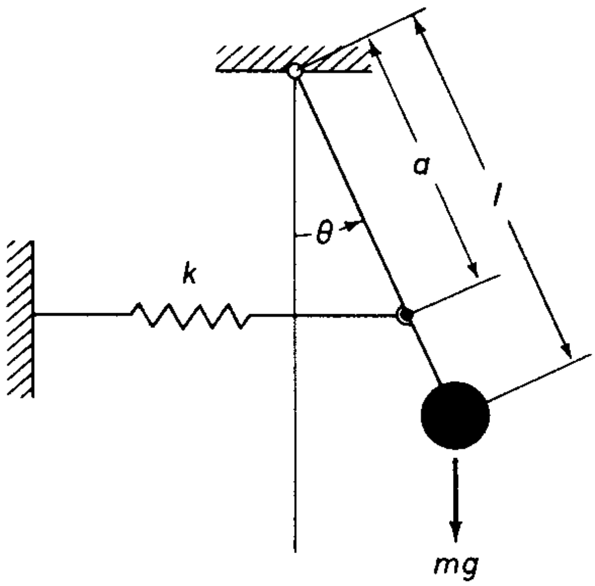
Problem 28:

Derive the equation of motion for the pendulum system shown and obtain the natural frequency. Assume that when the pendulum is vertical there is no spring force; also, assume that θ is small. Finally, determine $\theta(t)$ when the pendulum is given initial conditions $\theta(0)=\theta_0$ and $\dot{\theta}(0)=0$.



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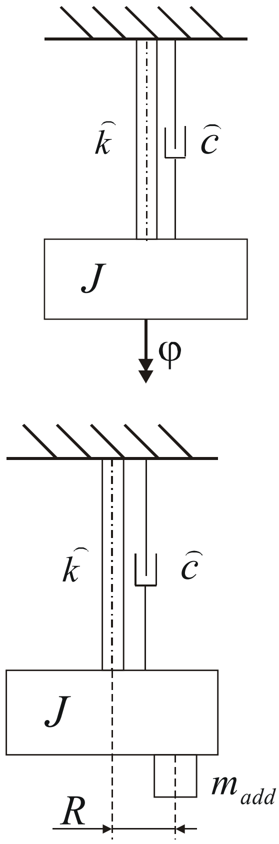
Problem 29:

Determine the stiffness $\widehat{k}$, the damping coefficient $\widehat{c}$ and the mass moment of inertia J of a one degree of freedom torsion oscillator using the following decaying oscillation curves with and without additional mass m_{add}.

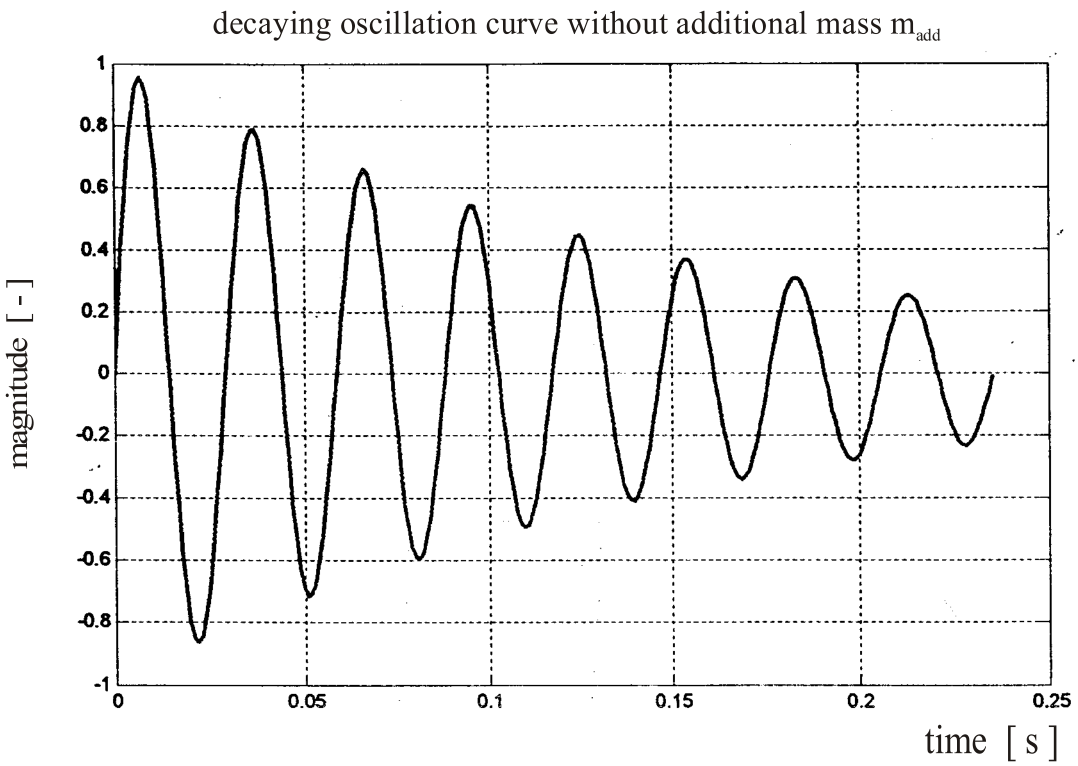


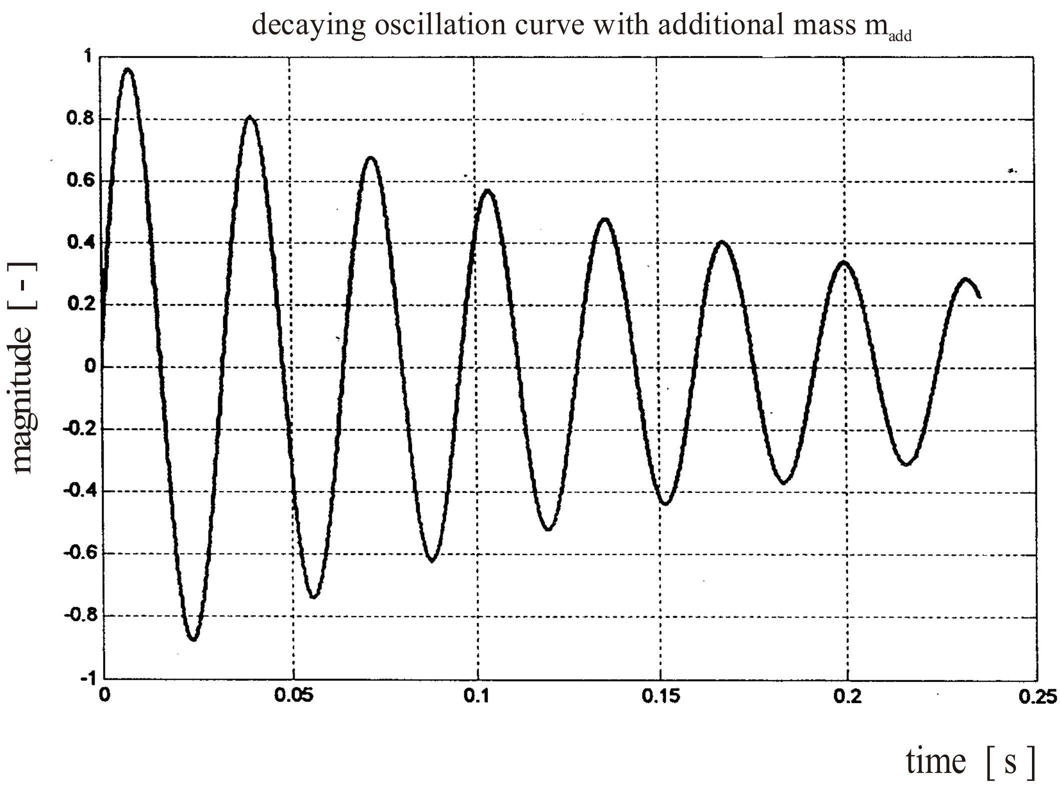
Given: $R = 54\ mm$

$m_{add} = 2\ kg$

$x = 98\ mm \mathrel{\hat{=}} 0{,}2\ s$ $\qquad y = 82\ mm \mathrel{\hat{=}} 2$

1. Determine the mean value of the damping ratio D
2. Determine the time of a cycle T and T_{add} with and without additional mass, respectively.
3. Determine the stiffness $\widehat{k}$, the damping coefficient $\widehat{c}$ and the mass moment of inertia J.


decaying oscillation curve without additional mass m_{add}
magnitude [-]
time [s]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.05
0.1
0.15
0.2
0.25


decaying oscillation curve with additional mass m_{add}
magnitude [-]
time [s]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.05
0.1
0.15
0.2
0.25

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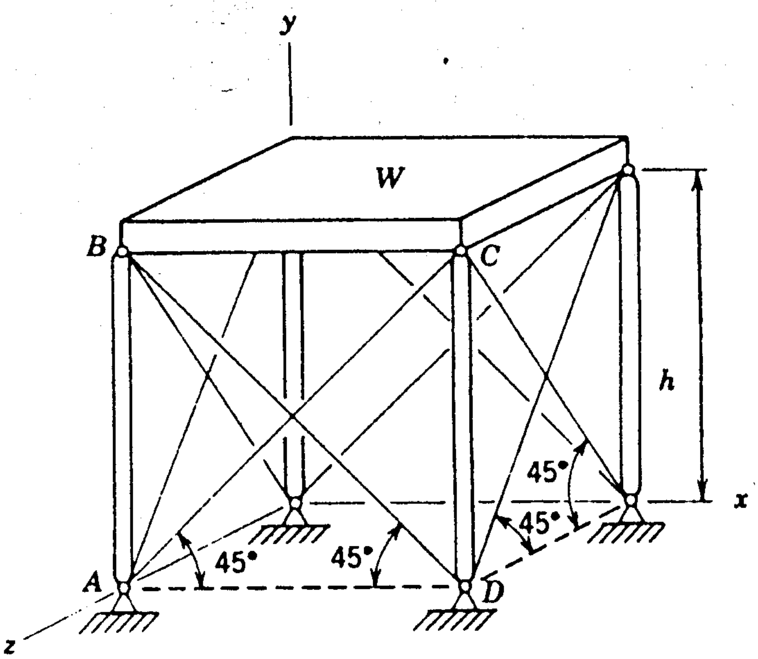
Problem 30:

A simple frame structure consists of a rigid plate with mass $m = 13{,}8\,t$ which is supported by four vertical rods. Two wire ropes are stretched between two vertical rods that fix the system in its equilibrium position. The two steel wire ropes have a cross-sectional area of $A = \frac{\sqrt{2}}{2}\,mm^2$ and are powerful pre-stretched. Neglect all masses apart from the mass of the plate to determine the time of a period T of a horizontal oscillation.

Given: $m, A, h = 10\,m, E = 210\,\frac{kN}{mm^2}$



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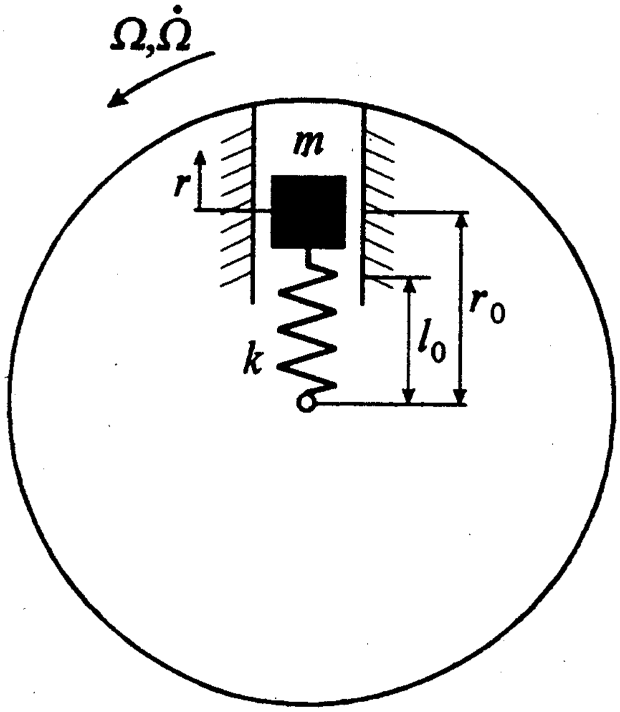
Problem 31:

A guided mass on a rotating disk is fit to a spring with stiffness k. The distance between the mass and the centre of the disk is l_0 if the disk stops. The spring is uncompressed in this case.



a) Determine the eigenfrequency ω_0 of the spring-mass-system if the disk stops.
b) Determine the static equilibrium position $r_0(\Omega)$ of the mass m, if the disk rotates with constant angular speed Ω.
c) Assume now small oscillations about the already determined static equilibrium position. Determine the equation of motion of the system. Determine the eigenfrequency of the rotating system.
d) What happens if the angular speed Ω of the disk is greater than the eigenfrequency ω_0 of the system?

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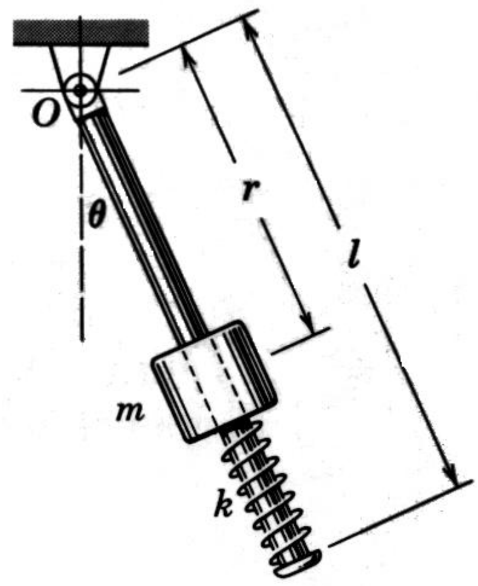
Problem 32:

The small cylindrical mass m is confined to slide on the slender rod of length l and mass m_0 which is free to rotate in the vertical plane about O. The spring which supports the mass m has a stiffness k, and $r = r_0$ when it is uncompressed. Write the differential equations of motion corresponding to the two degrees of freedom of the system. Neglect friction and the mass of the spring. Use Lagrange's Equation.



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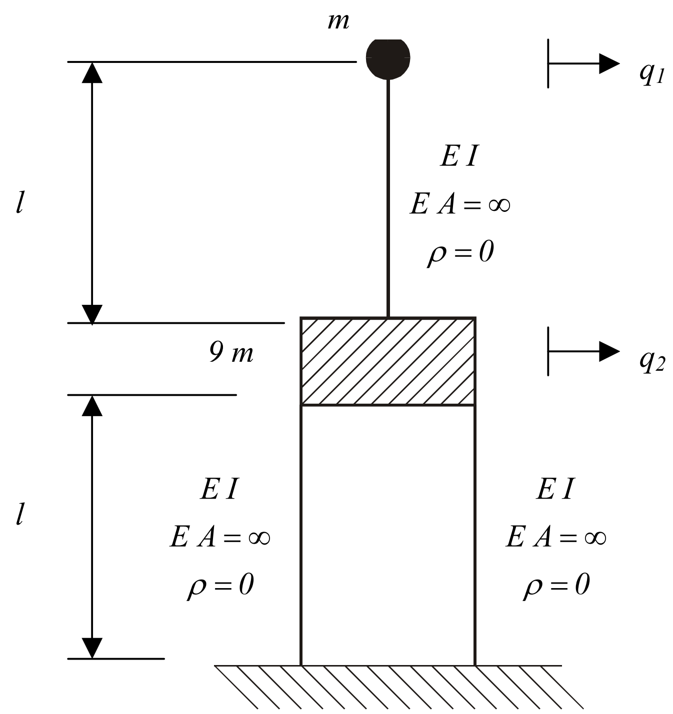
Problem 33:

1. Determine the Equation of Motion of the drawn oscillator with coordinates q_1 and q_2.
2. Determine the eigenfrequencies and eigenvectors.
3. Figure the mode shapes.

Hint: $K = \frac{EI}{l^3}$ $\omega_0 = \sqrt{\frac{K}{m}}$



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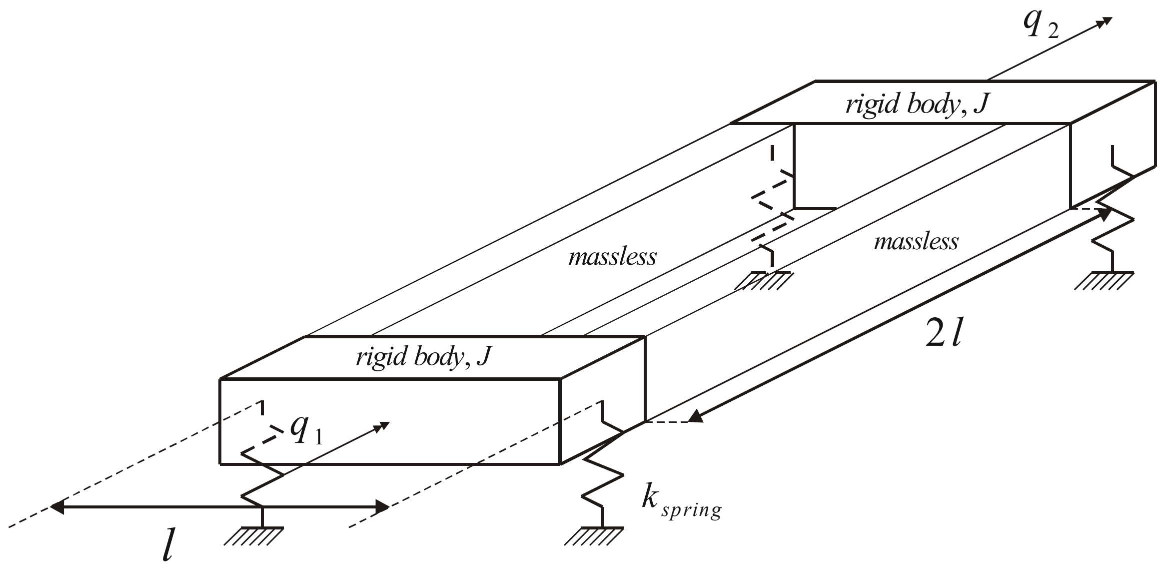
Problem 34:

Determine the eigenfrequencies and eigenvectors of rotation. The frame structure consists of two parallel and massless beam elements with stiffness k_{beam} and two rigid bodies with mass moment of inertia J about the q-axis. The frame structure is simply supported by four symmetric spring elements.

1) Determine the stiffness matrix $\underline{\underline{K}}$ and the mass matrix $\underline{\underline{M}}$.
2) Determine the eigenfrequencies and eigenvectors.
3) Draw the mode shapes.
4) Is it possible to manipulate the first eigenfrequency by changing the stiffness of the beam elements?



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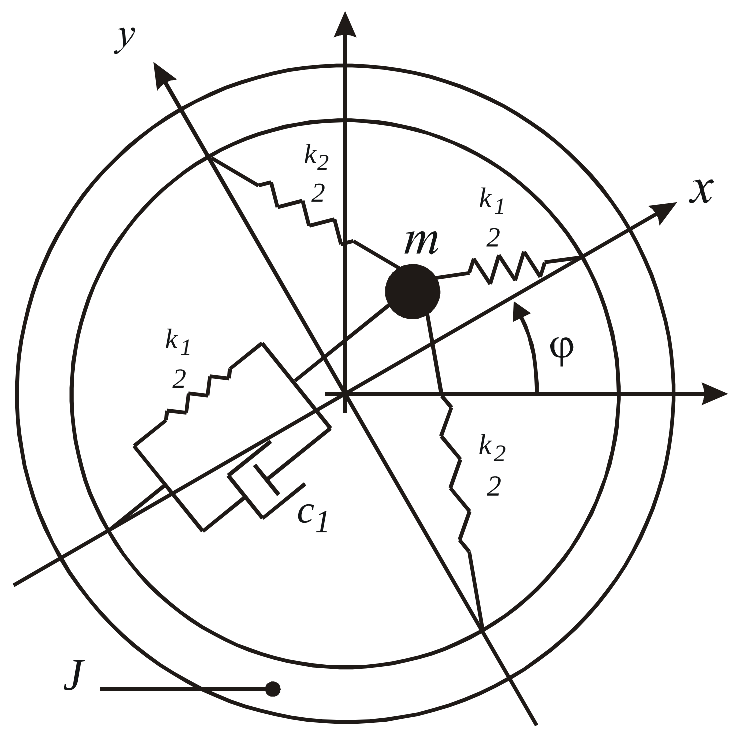
Problem 35:

Determine the equations of motion of the point mass *m* inside the rotating ring. Use Lagrange's Equations of Motion.



x, y: rotating coordinate system
$\Omega = \dot{\varphi} = const.$
small deflections