UNIVERSITY OF SIEGEN
DEPARTMENT OF ELECTRICAL ENGG. AND COMPUTER SCIENCE

Institute of Control Engineering — Prof. Dr.-Ing. R. Mayr

Advanced Control

Exercise 1

1. Task:

   Transform the differential equations of higher order into corresponding systems of differential equations of 1st order and set up the state equations.

   a) $\ddot{y}(t) - 6\dot{y}(t) + 4y(t) = 3u(t)$

   b) $\ddot{y}(t) + 2y(t) = u(t)$

      How affects the missing first derivative of y(t) on the left side of the differential equation on the evolution matrix?

   c) $\dddot{y}(t) = u(t)$

   d) $\ddot{y}(t)\,\dot{y}(t) + 4y(t) = 3u(t)$

2. Task:

   The following system is given in state space description

$$\underline{\dot{x}}(t) = \begin{bmatrix} a_{11} & 3 \\ 4 & -6 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \underline{x}(t) + d \quad u(t)$$

   a) Which class of systems lies ahead?
      - time variant, time invariant?
      - linear, non-linear?
      - Single input/output system, multivariable system?

   b) Of which order the system?

   c) Transform the system for the special case $a_{11} = 0$, $b_1 = 0$, $c_2 = 0$, $d = 0$ into a differential equation of higher order. Remain in the time domain while transforming.

   d) Now regard the more general case with $a_{11} \neq 0$, $c_2 \neq 0$ and $d \neq 0$, while $b_1 = 0$ may still exist. For which reason, the transformation into a system of higher order becomes difficult, if it is exclusively carried out in the time domain?

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Advanced Control

Exercise 2

1. Task:
   The system known from the previous exercise is given in state space description

$$\dot{\underline{x}}(t) = \begin{bmatrix} a_{11} & 3 \\ 4 & -6 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ b_2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \underline{x}(t) + d \quad u(t)$$

a) Perform for the general case the transformation into a system of differential equations of higher order using the Laplace transformation.
b) Which role to plays here the value for d?
c) Verify the result by a comparison with the result from the previous exercise.

2. Task:
   The following system is given in state space description

$$\dot{\underline{x}}(t) = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \underline{u}(t)$$

$$\underline{y}(t) = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 & 0 \\ 0 & d_{22} \end{bmatrix} \underline{u}(t)$$

a) Which is the class of systems lies ahead?

b) Calculate the complex transfer matrix $\underline{G}(s)$.

c) Interpret in which way the input variables affect the output variables each.

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Advanced Control

Exercise 3

1. Task:
Two linear timeinvariant single input/output systems are given:
1st system

$$\dot{\underline{x}}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & 4 \end{bmatrix} \underline{x}(t)$$

2nd system

$$\dot{\underline{x}}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 4 \end{bmatrix} \underline{x}(t)$$

a) Answer without further calculations the question about controllability and observability for both systems and give a short explanation.

b) Check your answer given in a) by the corresponding calculations.

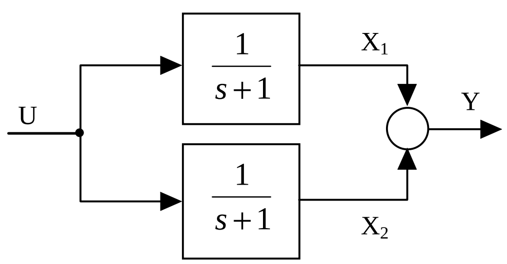
2. Task:
The following system is given with the complex transfer function

$$F(s) = \frac{s^3 + 4s^2 + 2}{s^3 + 3s^2 + s}$$

a) Transform this system to the controllability canonical form.

b) Is the system controllable ?

3. Task:
Is the following system drawn in the flow diagram controllable and/or observable?
Give an explanation!



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Exercise 4

1. Task:

Transfer the system represented by the complex transfer function

$$F(s) = \frac{(s^2 + 3s + 2)(s + 4)}{(s + 1)^2 (s + 3)^2}$$

into the Jordan canonical form.

2. Task:

The system represented in state space description

$$\dot{\underline{x}}(t) = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 3 & 1 \end{bmatrix} \underline{x}(t)$$

must be transferred by the means of the transformation matrix

$$\underline{T} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

into an equivalent system.

a) Calculate the system equations

$$\dot{\underline{\bar{x}}}(t) = \underline{\bar{A}}\,\underline{\bar{x}}(t) + \underline{\bar{b}}\,u(t)$$

$$\underline{y}(t) = \underline{\bar{c}}^T \underline{\bar{x}}(t) + \bar{d}\,u(t)$$

for the equivalent system.

b) How can the transformed state variables $\bar{x}_1(t)$, $\bar{x}_2(t)$ be computed from the original state variables $x_1(t)$, $x_2(t)$?

c) Provide the proof that the original system as well as the transformed system have the same transmission properties.

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<u>Advanced Control</u>

Exercise 5

1. Task:

The linear time invariant system of 2nd order is given

$$\underline{\dot{x}}(t) = \begin{bmatrix} -1 & 0 \\ -2 & -3 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 2 & 3 \end{bmatrix} \underline{x}(t)$$

Find the solution for this system while transforming it into the form, which is similar to the Jordan Canonical Form. Then find the matrix-e function.

The initial time $t_o = 0$ and the initial state $\underline{x}(t_o) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

2. Task:

For the system

$$\underline{\dot{x}}(t) = \begin{bmatrix} -2 & 4 \\ -9 & -2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \underline{x}(t) ,$$

the solution y(t) has to be found. The initial state is

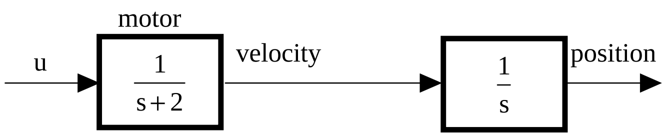
$$x(t_0 = 0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The system input is u(t)=0.

The matrix $e^{\underline{A}t}$ shall be found by a short term transformation into the frequency domain.

3. Task:

For a motor, a position control system with pole placement is to be projected.



a) Transfer the system into state space description.
b) Find the gain vector $\underline{r}^T$ for the state feedback. The overall system should have one pole at -1,0 and one pole at -1,5.
c) Outline the corresponding flow diagram.
d) Which value for the gain in the nominal branch do you propose?

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Exercise 6

1. Task:

The following system

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}(t)$$

with the feedback vector $\underline{r}^T = [1.5\ 0,5]$ is given.

a) Find the corresponding value for the input gain r_W , which fits here, while analysing the corresponding general equation.

b) Describe the behavior of the overall system in state space description.

c) Determine the complex transfer function $F_W(s)$ for the overall system.
Where are the poles of $F_W(s)$ located?

d) Evaluate the response of the overall system by the means of the steady state theorem of the Laplace transform for $t \to \infty$ with the input realized by the step function $w(t) = \sigma(t)$.

2. Task:

For the system

$$\dot{\underline{x}}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & 2 \end{bmatrix} \underline{x}(t)$$

a controller with complete state feedback should be designed.

a) Transfer this system into the controllability canonical form. The corresponding transformation matrix is

$$T = \begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{2} & \frac{2}{3} \end{bmatrix}$$

b) For the resulting system in the controllability canonical form find a state feedback control $\underline{\bar{r}}^T$. The poles for the overall system should be located at $s_1 = -3$ and $s_2 = -4$. Find also the corresponding value for the input gain r_W .

c) From here, evaluate the state feedback for the original system, which is stated above. Find also the corresponding input gain for the original system.

3. Task:

For the system

$$\dot{\underline{x}}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & 3 \end{bmatrix} \underline{x}(t),$$

a controller with complete state feedback shall be developed.

The eigenvalues of the overall system should be located at $\lambda_1 = -10$ and $\lambda_2 = -15$.

a) Evaluate the state feedback vector $\underline{r}^T$.

b) Give an explanation of the solution.

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Exercise 7

1. Task:

For the motor

$$\underline{\dot{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}(t)$$

an observer is to design.

a) Interpret the physical relevance of this observer.
b) Perform the design of the observer. The eigenvalues of the estimating error differential equation should be located at $\lambda_1^B = -20$ and $\lambda_2^B = -30$.
c) Draw the corresponding flow diagram.
d) Find the state differential equation for the system equipped with the observer. Which eigenvalues are present here?
e) Add a control system with complete state feedback. That feedback gain vector is $\underline{r}^T = [r_1 \quad r_2]$, the input gain in the nominal branch is r_w.
f) The transfer function of overall system, which is equipped with the observer as well as with the state feedback controller is $F_w(s)$.

   In the steady-state the step response is: $\lim\limits_{s \to 0} s \cdot F_w(s) \cdot \frac{1}{s} = 1$.

   Indicate a possibility, to increase the possible robustness of this control system with regard to possible disturbances.

2. Task:

For the multivariable system (2 input- and 2 output variables)

$$\underline{\dot{x}}(t) = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \underline{u}(t)$$

$$\underline{y}(t) = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix} \underline{x}(t)$$

a decoupling control system should be designed.

a) Draw the flow diagram of the controlled system.
b) Design a decoupling controller for the multivariable system. Consider the parameters $\alpha_{10} = l_1$ and $\alpha_{20} = l_2$.
c) Draw the flow diagram only of the controller.
d) Draw the flow diagram that is reflecting the dynamic behaviour of the overall system.